

## Novel multi-beam spring design for vibration energy harvesters

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### Abstract

Vibration energy harvesters are transducers that are used to convert the ambient vibrations into electrical energy to power the sensors. Vibration energy harvesters rely on the fact maximum power is produced when their natural frequencies are in resonance with the source vibrations. Since the ambient vibrations are random in nature there is a necessity to develop broadband vibration energy harvesters that are capable producing maximum power at least to the prominent frequencies from the random vibration source. A novel design of a multi-beam vibration energy harvesting structure is proposed that is capable of generating maximum power at multiple source frequencies. An analytical model for the proposed design was first developed and then validated with the numerical model.

**Keywords:** Vibration energy harvester, multi-beam vibration, spring stiffness, Fast Fourier Transform, lumped mass.

### 1 Introduction

Energy harvesting is one of the prerequisite requirement to make the sensors ubiquitous since the sensors have to power themselves from the ambient power sources. Energy harvesters are transducers which convert the ambient sources of energy into electrical energy that will suffice to power the sensors [1]. In the internet of things (IOT) applications, irrespective of whether the sensors are active or passive, energy is still required for transmitting the sensed data to peer sensors or the wireless routers [2]. This is not possible to be solved by batteries which get exhausted rendering the wireless sensor node inactive. This challenge has developed the field of energy harvesting research.

Energy harvesters are of various types depending on the ambient sources they scavenge upon. predominantly, they are categorized as vibration energy harvesters, thermal energy harvesters, chemical energy harvesters and light energy harvesters [3]. Amongst these, vibration energy harvesters which scavenge energy from the ambient vibration is useful for sensors that are in the vicinity of vibration producing machinery such as rotating machinery (generators and motors). One such state-of-art application is the use of vibration energy harvester in conjunction with the GE Insight mesh wireless sensors

network at Bently Nevada Shell plant for wireless remote condition monitoring [4].

One of the essential requirement of the vibration energy harvesters is to generate energy continuously so that there is continuous operation of sensing, transmitting the data, receiving the peer data. Vibration energy harvesters rely on the fact that maximum power is obtained when the source vibration is in resonance with the natural frequency of the energy harvester. This is highly challenging for energy harvesting research since the source vibrations are random in nature characterized by multiple frequencies. This has necessitated the research on developing transduction mechanisms that have multiple resonance structures such as varied lengths of cantilevers.

In the proposed concept, a multi-beam energy harvesting structure is proposed which consists of equal length beams with their fixed support ends and with lump masses placed on each of them at varied distances from the left reference plane. The analytical model and the numerical finite element analysis model were developed and simulated for different beam thicknesses.

### 2 Concept design

The concept design of the multi-beam energy harvesting structure is shown in the Figure 1. The energy harvesting structure consists of beams 1 to 4. The lengths of all the four beams are equal with each of the beams equal to  $l = 125\text{mm}$ . The four equal lump masses  $m_1, m_2, m_3$  and  $m_4$  are placed at  $p_1 = 25\text{mm}$ ,  $p_2 = 50\text{mm}$ ,  $p_3 = 75\text{mm}$  and  $p_4 = 100\text{mm}$ . The corresponding lengths  $q_1, q_2, q_3$  and  $q_4$  are  $100\text{mm}$ ,  $75\text{mm}$ ,  $50\text{mm}$  and  $25\text{mm}$  respectively such that  $p_i + q_i = l$  where  $i = 1, 2, 3, 4$ .

### 3 Analytical model

The generic equation for the equivalent spring stiffness of a beam with two fixed ends at a distance  $x$  from the left-hand side fixed support is given by,

$$(x) = \frac{6EI l^3}{(l-x)^2 p^2 (3ql - (1-p)(3q+p))} \quad (1)$$

where  $E$  = modulus of elasticity,  $I$  = area moment of inertia of the cross section of the beam and  $l$  = length of

the beam and is divided into two parts  $p$  and  $q$ . For the proposed design, the distance  $x = p_i$ , for  $i = 1$  to 4 for the four beams and hence equation 1 can be expressed as equation 2 and equation 3.

$$k_i = k(p_i) = \frac{6EI l^3}{q_i^2 p_i^2 (3q_i l - q_i(3q_i + p_i))} = \quad (2)$$

and,

$$k_i = k(p_i) = \frac{3EI l^3}{(p_i q_i)^3} = \frac{Q}{(p_i q_i)^3}, i = \forall 1, 2, 3, 4 \quad (3)$$

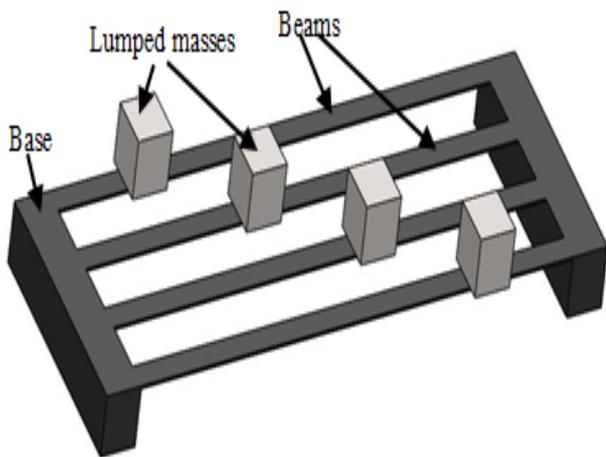
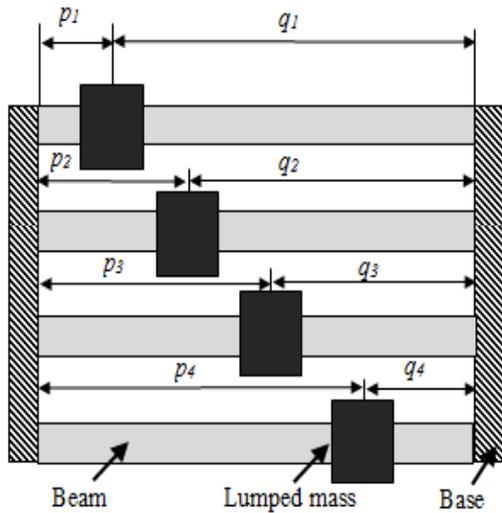
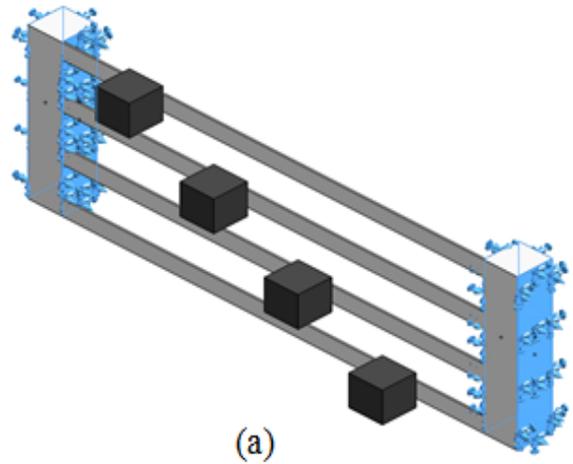


Figure 1: Multi-beam vibration energy harvesting structure.

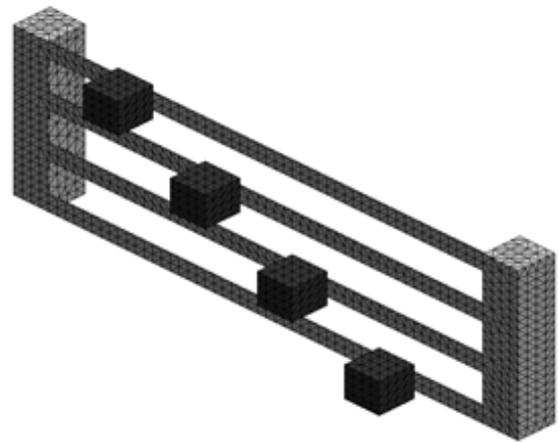
In equation 2,  $Q = 3EI l^3$  is a constant for this design since the length( $l$ ), breadth( $b$ ) and the thickness( $t$ ) are same for all the four cantilevers considered for each set of trials. Three trials were conducted by varying the beam thicknesses at  $t = 0.5\text{mm}$ ,  $1\text{mm}$  and  $2\text{mm}$  which varied the second moment of inertia ( $I$ ) for determining  $k_e$  and the effective mass ( $m_e m + m_b/3$ ), where lumped mass ( $m$ ) and the mass of the beam ( $m_b$ ).

## 4 Numerical simulation

The numerical finite element analysis simulation was performed on the multi-beam 3D model by fixing the fixed supports as shown in the Figure 2 and by applying a displacement of 3mm along the direction of vibration. The 3D model was meshed with solid standard tetrahedral mesh of 9846 elements and 18094 nodes with an average element size of 2.51mm.



(a)



(b)

Figure 2: (a) Fixtures and loads and (b) meshed model of Multi-beam vibration energy harvesting structure.

Similar to the analytical simulation, three trials for thicknesses  $t = 0.5\text{mm}$ ,  $1\text{mm}$  and  $2\text{mm}$  was performed to study the resonant frequencies and mode shapes of the beams. From the proposed design of placing the lumped masses at varied distances for each of the four beams, it was anticipated that at resonant frequencies of each mode correspond to one of the beams.

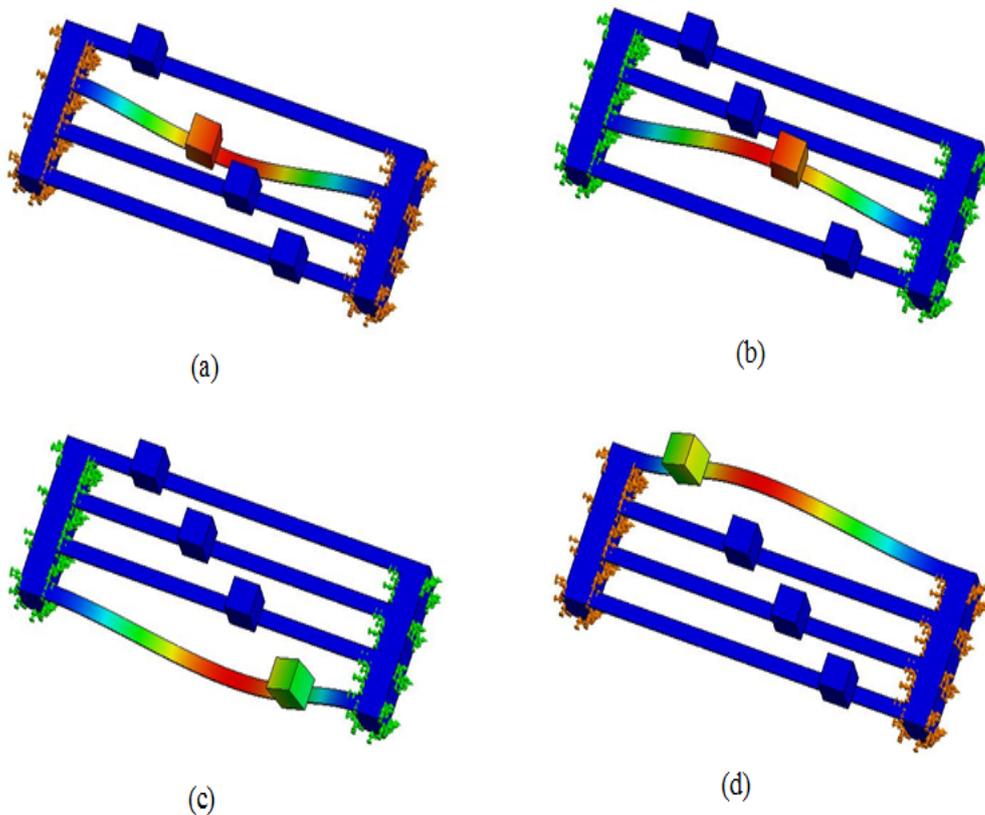


Figure 3: Numerical values of maximum displacements of multi beam VEH at (a) first mode (b) second mode (c) third mode and (d) fourth mode.

## 5 Results and discussion

The maximum displacements for the first four modes of vibration of the beams when excited at an amplitude of 3 mm is as shown in the Figure 3. It can be observed that the maximum mass participation was along the direction of vibration at all the first four modes. It can also be observed that vibration at the first and the second mode excites beams 2 and 3 at 8.45 Hz while beams 2 and 4 have maximum displacement during third and the fourth modes at 13.6 Hz. This was intended in the design to achieve broadband energy harvesting.

The numerical values of stiffness  $k(p_i)$  was determined by  $\omega(p_i)$  obtained from the FEA simulation. Since the analytical simulation considered only the beam and not the superstructure, the cumulative effective mass participation at the direction of vibration was considered for calculating the numerical  $k(p_i)$ . The analytically and the numerically obtained values of  $k(p_i)$  and  $\omega(p_i)$  is as shown in the Table 1. It should be understood that while analytical calculations were done using a single fixed beam, the numerical FEA considered the entire multi-beam structure and this led to the difference in  $\omega(p_i)$  values.

The maximum percentage error is 29.15% between the

analytical and numerical values is due to the cumulative effective mass participation factor being approximately around 92% in the direction of vibration. The trend of both the analytical and numerical values  $k(p_i)$  with respect to the distance of the lump mass placement  $p_i$  and the thickness variation of the beams agree with each other validating the model.

The current design can be generalized for  $n$  number of beams where  $i = 1, 2, \dots, n$  where the lumped masses are diagonally placed equidistant to each other as shown in the Figure 4(a). From Table 1 it can be generalized that  $(1, n), (2, n-1), (3, n-2) \dots (n/2, n/2)$  have more or less same resonant frequencies. Therefore, only half the number of the beams are sufficient by considering only  $1, 2, \dots, n/2$  beams to get the same range of frequencies thus reducing the excess beams as shown in Figure 4(b). The asymmetric structure as shown in Figure 4(b) can be fabricated in multitude at MEMS scale as shown in Figure 4(c). If a designer wants to develop a MEMS energy harvester for an application (say motor condition monitoring) then one has to perform a Fast Fourier transform and detect the prominent five frequencies. Then cell structures are fabricated at MEMS scale such that each beam within the cell vibrates at a particular modal resonance.

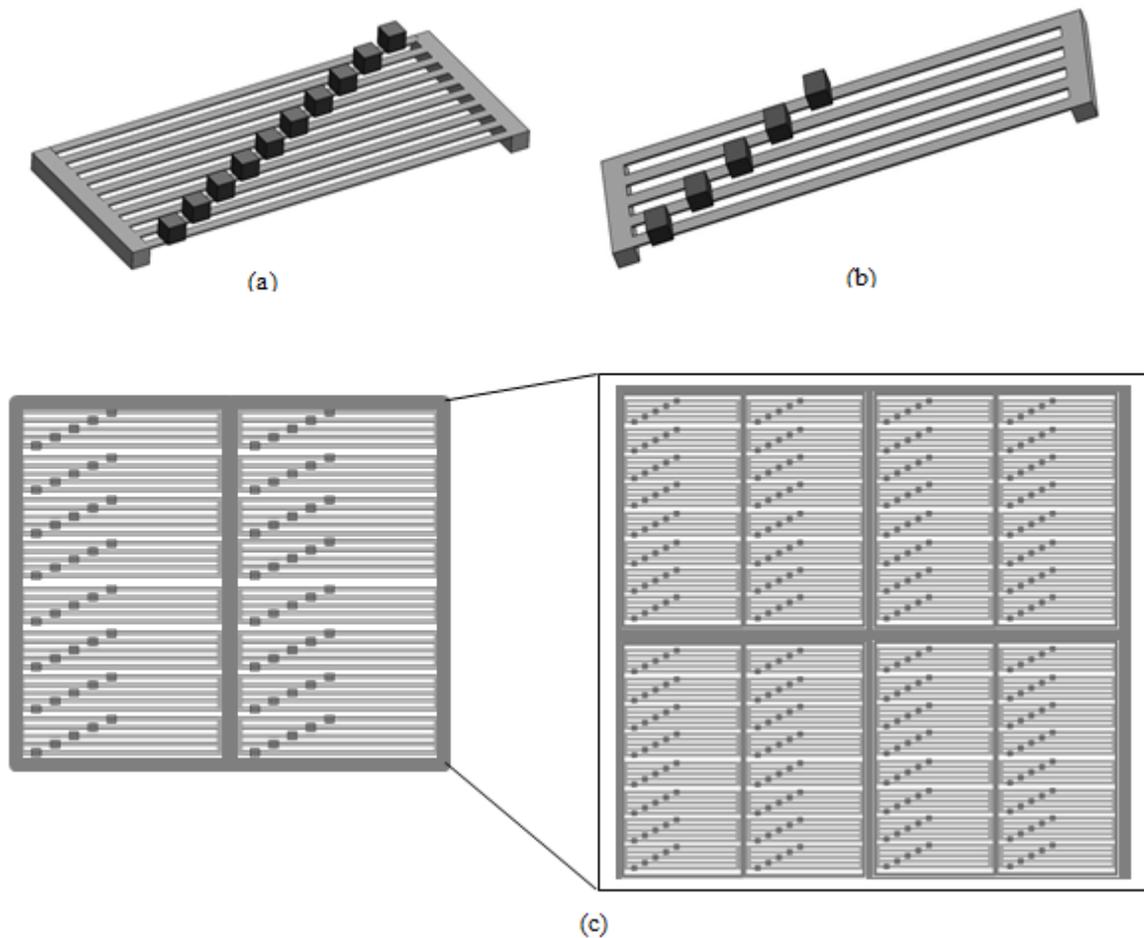


Figure 4: Generalized multi-beam vibration energy harvester model (a) symmetric scheme (b) non symmetric scheme (c) MEMS scale schematic.

Table 1: Comparison of analytical and numerical values of stiffnesses and resonant frequencies.

Thickness of the beam (h), (mm)	$p$	Analytical			Numerical (FEA)			Error % in $k(p_i)$
		$k(p_i)$	$\omega(p_i)$	$f(p_i)$ , (Hz)	$k(p_i)$	$\omega(p_i)^*$	$f(p_i)^*$ , (Hz)	
0.5	25	24	61.67	9.81	29.25	85.41	13.59	7.69
	50	8	33.57	5.34	11.29	53.07	8.45	29.15
	75	8	33.57	5.34	11.36	53.24	8.47	29.6
	100	27	61.67	9.81	29.12	85.22	13.56	7.27
1	25	216	174.42	27.76	206.37	226.87	36.11	4.67
	50	64	94.94	15.11	83.79	144.56	23.01	23.62
	75	64	94.94	15.11	84.09	144.82	23.05	23.62
	100	216	174.42	27.76	208.23	227.89	36.11	3.73
2	25	1728	493.34	78.52	1352.15	580.72	92.42	27.81
	50	512	268.54	42.74	602.36	387.60	61.69	15.01
	75	512	268.54	42.74	605.26	388.53	61.84	15.41
	100	1728	493.34	78.52	1357.04	581.77	92.59	27.34

The variation of the stiffnesses ( $k(p_i)$ ) and the  $\omega(p_i)$  with the distance of the lumped mass placement  $p_i$  and the thickness  $h$  is as obtained as shown in the Figure 5. For optimized performance, the energy harvester can be tailor made using the analytical model as given in equation 3 by tuning the design to the prominent frequencies as determined by Fast Fourier Transform (FFT) analysis of the source vibrations.

For a  $n$  numbered multi-beam vibration energy harvester, the natural frequency of individual beams for

different values of  $p_i$  was observed to have a bath-tub curve as in Figure 5. The common horizontally projected spacing between the lumped masses ( $s$ ) determines the relation between  $p_i$  and  $q_i$  and also the number of natural frequencies achievable for a given design. The spacing ( $s$ ) was varied from 1mm, 5mm, 10mm and 15mm and 20mm respectively. At  $s = 1$ mm, one can observe the symmetric bath tub curve which is as anticipated from the inference of Table 1 of going ahead with the asymmetrical design as shown in Figure 4(b).

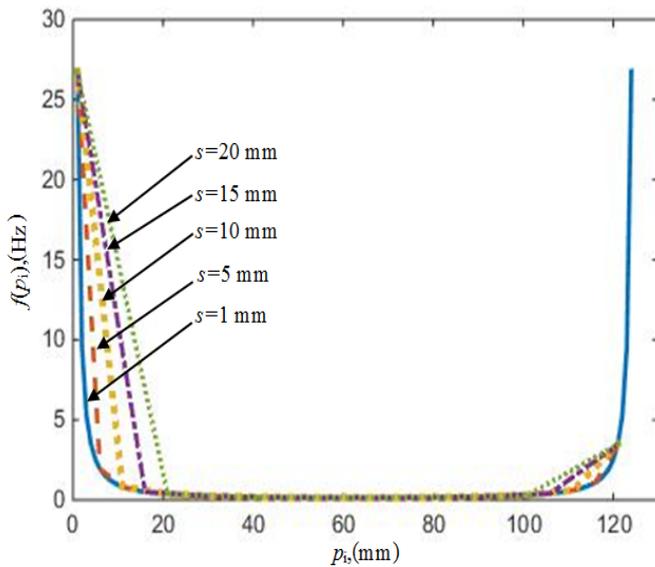


Figure 5: Variation of the stiffnesses ( $k(p_i)$ ) and their natural frequencies  $f(p_i)$  for spacing schemes  $s = 1, 5, 10, 15$  and  $20$ mm.

## 6 Conclusion

Vibration energy harvesters are prominently used in machine condition monitoring applications due to the presence of ambient vibrations produced by the machines during operation. Since vibration energy harvesters depend on the resonant frequencies for maximum power, their structure should be designed such that the inertial

parts get excited for each random frequency of the source vibrations. A novel multi-beam vibration energy harvester structure is proposed where the lumped masses are diagonally placed at different distances such that each mode corresponds to each beam. The analytical model of this structure was numerically validated with accuracy of 29%. Whilst this is appreciable, further investigations are necessary together with experimental validation to understand and validate the model. This model can be applied for developing MEMS scale inertial mass based energy harvesting structures.

## References

- [1] R. R. Gatti and I. M. Howard, "Electromagnetic Energy Harvesting by Spatially Varying the Magnetic Field", In Proceedings of the 2011 2<sup>nd</sup> International Congress on Computer Applications and Computational Science, vol. 1, pp. 403, Springer Science & Business Media, February, 2012.
- [2] M. Gorlatova, J. Sarik, G. Grebla, M. Cong, I. Kymissis and G. Zussman, "Movers and shakers: Kinetic energy harvesting for the internet of things", In ACM SIGMETRICS Performance Evaluation Review, vol. 42, no. 1, pp. 407-419, June, 2014.
- [3] S. Priya and D. J. Inman, "Energy harvesting technologies", New York: Springer, vol. 21, Eds, 2009.
- [4] General Electric 2015 Bently Nevada- Essential Insight.mesh- Wireless Condition Monitoring GE Measurement Control GEA15019E - 01/2015